

# A SIGNAL PROCESSING SCHEME FOR REDUCING THE CAVITY PULLING FACTOR IN PASSIVE HYDROGEN MASERS

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## ABSTRACT

A passive hydrogen maser operates so as to cause a signal frequency,  $f_s$ , to satisfy a selected criterion. The frequency  $f_s$  which satisfies the criterion depends on the cavity resonance frequency,  $f_c$ . The derivative of  $f_s$  with respect to  $f_c$  for  $f_c = f_s$  is called the PULLING FACTOR. Theoretically this factor can be zero with a computational criterion making use of complex signal voltage samples taken at several frequencies.

## INTRODUCTION

Consider the oscillator control servo for a passive hydrogen maser. A signal frequency  $f_s$  is synthesized from the oscillator output frequency. The oscillator frequency is controlled so as to cause  $f_s$  to equal  $F_0$ , the (perturbed) hydrogen transition frequency. Possible servo control criteria include:

- a. The difference in maser transfer magnitudes at frequencies equally spaced above and below  $f_s$  equals zero, the frequency spacing being less than the hydrogen linewidth.
- b. The maser transfer phase at frequency  $f_s$  equals zero.
- c. The difference between the maser transfer phase at  $f_s$  and the mean of the phases at frequencies equally spaced above and below  $f_s$  equals zero, the frequency spacing being very much larger than the hydrogen linewidth.

The signal frequency  $f_s$  which satisfies the selected criterion depends on the value of the cavity resonant frequency  $f_c$ . The derivative of  $f_s$  with respect to  $f_c$ , under servo control, is the PULLING FACTOR.

Pulling factors for criteria (a) and (b) above are derived in Section 11 of reference 1, and are substantially different for the two cases. For criterion (c) it can be shown that the pulling factor is closely the ratio of the cavity and hydrogen line Q's, also differing from criteria (a) and (b). It appears then that, at least in part, the pulling factor is a result of the method used to cause  $f_s$  to approximate the hydrogen transition frequency.

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The pulling factors mentioned above were all derived using a model of the steady state complex microwave field as a function of frequency given by Lesage, Audoin and Tetu in reference 1 (1979). Using the same model it is possible to devise a criterion for which the derivative of  $f_s$  with respect to  $f_c$  is zero when  $f_c = f_s$ . Unlike criteria (a), (b) and (c) above it makes use of both magnitude and phase of transfer measurements at  $f_s$  and at frequencies equally spaced above and below  $f_s$ .

#### PROPOSED DEMONSTRATION HARDWARE

At the present time the proposed frequency error criterion has not been demonstrated experimentally. An experiment should evaluate at least the following:

- Cavity pulling characteristic
- Frequency error noise due to receiver noise
- Bias due to error estimation algorithm.

Figure 1 is a simplified block diagram for demonstration of the signal processing scheme. Some functions such as digital/analog conversion and cavity tuning are not shown where needed. The process, under computer control, includes the following steps:

switch the maser input signal ( $V_1$ ) to frequencies  $f_a$ ,  $f_s$ , and  $f_b$  in sequence (FREQUENCY SYNTHESIZER)

measure complex voltage ratios at these frequencies (NETWORK ANALYZER)

acquire and filter complex samples (COMPUTER)

perform frequency error computations (COMPUTER)

perform servo loop filter functions (COMPUTER)

correct oscillator frequency error (COMPUTER).

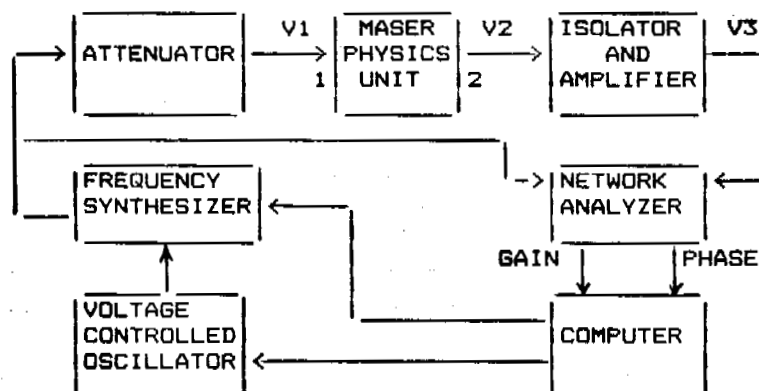


FIGURE 1 CONCEPTUAL BLOCK DIAGRAM

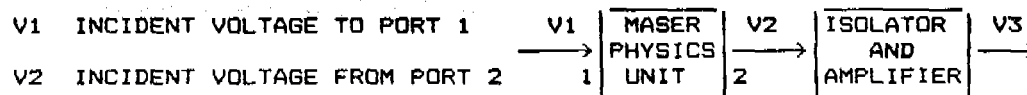
## CAVITY RESONATOR INVERSE TRANSFER FUNCTION

The steady-state complex transfer function of the passive maser is the ratio of the voltages  $V_2$  and  $V_1$  defined in Figure 2. For a high-Q single-pole cavity resonator without atomic hydrogen the function consists of a fraction with a constant numerator. The denominator is unity plus an imaginary term  $vc$  which is a linear function of frequency. For practical reasons Figure 2 includes an isolator and an amplifier whose output is  $V_3$ . We consider measurements of complex values of the ratio of  $V_3$  and  $V_1$ .

In equation (1) the symbol  $H_c$  is introduced, which is the ideal cavity transfer function denominator, Equation (2). In the definition of  $vc$  (3) we see that the imaginary term is equal to the twice the difference between the signal frequency and the cavity resonant frequency divided by the cavity bandwidth.

The complex plot of Figure 2 shows the contour of  $H_c$  as frequency is varied, with a particular value indicated by \*. The simplicity and linearity of  $H_c$  suggests the use of inverse transfer functions for parameter estimation from measurement data.

If we obtain an inverse transfer function from measured voltages as  $V_1 / V_3$  it will consist of  $H_c$  multiplied by a complex number  $H_0$  which results from cavity insertion loss, various phase shifts, and gains and losses associated with the paths from the measurement junctions to the maser. In the following development we assume that variation of  $H_0$  over the band of frequencies of interest is negligible.



FOR THE IDEAL CAVITY WITHOUT ATOMIC HYDROGEN:

$$(1) \quad V_3 / V_1 = 1 / H_0 H_c$$

$H_0$  COMPLEX CONSTANT TO ACCOUNT FOR CAVITY INSERTION LOSS, AMPLIFIER GAIN, VARIOUS PHASE LAGS

$$(2) \quad H_c = 1 + j vc$$

$$(3) \quad vc = 2 (f - f_c) / B_c$$

$f$  FREQUENCY, HERTZ

$f_c$  CAVITY RESONANT FREQUENCY

$B_c$  CAVITY HALF-POWER BANDWIDTH

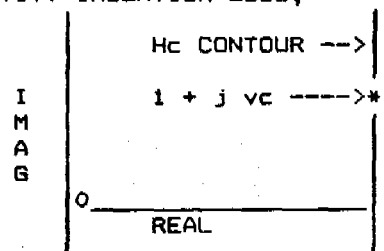


FIGURE 2 CAVITY RESONATOR INVERSE TRANSFER FUNCTION

Although  $H_0$  is unknown in the practical measurement situation, it is possible to find (estimate) its complex value and other parameters from measurements of  $V_1/V_3$  at two frequencies whose difference is known. Let the two frequencies  $f_a$  and  $f_b$  be equally spaced above and below a frequency  $f_s$  as illustrated in Figure 3. We will call the measured inverse transfer at these frequencies  $H_a$  and  $H_b$ , respectively. The corresponding values of  $H_c$  will be  $H_{ca}$  and  $H_{cb}$ , as in the table.

In equation (4) we take the mean of  $H_a$  and  $H_b$ , which is  $H_0$  times the mean of  $H_{ca}$  and  $H_{cb}$ , and find the product  $H_0 H_{cs}$  where  $H_{cs}$  is the value of  $H_c$  at frequency  $f_s$ , to be used later. From the difference between  $H_a$  and  $H_b$  we can find  $H_0$  multiplied by an imaginary constant, as in (5). We will use this expression later to remove the angle rotation due to  $H_0$  from calculations.

Dividing (4) by (5) produces an estimate of  $H_{cs}/(j^2 F_1 / B_c)$ . The real part of this quotient (6) is the difference between  $f_s$  and the cavity resonance frequency  $f_c$ , divided by  $F_1$  (known). The imaginary part (7) is the cavity bandwidth divided by  $2 F_1$ . Multiplying (5) and (7) and  $+j$  gives the complex value for  $H_0$ , as in (8). Hence two complex ratios suffice to characterize the ideal cavity resonator and the measured path:

- $H_0$  the inverse complex gain of the path including the cavity insertion loss
- $B_c$  the cavity bandwidth ( $f_c$  divided by loaded  $Q$ )
- $f_s - f_c$  the cavity tuning error in Hertz, if  $f_s$  is the desired resonant frequency.

$f_s$	SIGNAL FREQUENCY, VARIABLE	<-- $F_1$ --> <-- $F_1$ -->
$F_1$	SPACING FREQUENCY, CONSTANT	$f_b$ $f_s$ $f_a$ $f$

FREQUENCY	$V_1 / V_3$	$H_c$
$f_a = f_s + F_1$	$H_a = H_0 H_{ca}$	$H_{ca} = 1 + j^2 (f_s + F_1 - f_c) / B_c$
$f_b = f_s - F_1$	$H_b = H_0 H_{cb}$	$H_{cb} = 1 + j^2 (f_s - F_1 - f_c) / B_c$

$$(4) \quad (H_a + H_b)/2 = H_0 (1 + j^2 (f_s - f_c) / B_c) = H_0 H_{cs}$$

$$(5) \quad (H_a - H_b)/2 = H_0 (j^2 F_1 / B_c)$$

$$(6) \quad \text{REAL} \left( (H_a + H_b)/(H_a - H_b) \right) = (f_s - f_c) / F_1$$

$$(7) \quad \text{IMAG} \left( (H_a + H_b)/(H_a - H_b) \right) = -B_c / 2F_1$$

$$(8) \quad (-j B_c / 2 F_1) (H_a - H_b) / 2 = H_0$$

FIGURE 3 CAVITY SIDE-FREQUENCY TRANSFER RELATIONSHIPS

## MASER INVERSE TRANSFER FUNCTION

The passive maser transfer function with atomic hydrogen present can be readily derived from steady-state microwave field equations (35) through (38) from the paper of Lesage, Audoin, and Tetu in the Proceedings of the 33rd Annual Symposium on Frequency Control, 1979, pages 515 through 535 (reference 1). They assume that the cavity mistuning is small, and that the difference between the microwave frequency and the cavity resonant frequency is a small fraction of the cavity bandwidth.

Equation (9) introduces the symbol  $H_m$  for the denominator of the maser transfer function, multiplied by  $H_0$  as before. Equation (10) gives the function  $H_m$ , consistent with the field equations of reference 1, although different in appearance. The symbols  $\alpha$ ,  $S$ , and  $T_2$  are used as defined in reference 1.

The denominator of the maser transfer function is the sum of the terms presented above for the cavity transfer and a complex term due to the hydrogen atoms. The hydrogen contribution to the inverse transfer function is proportional to the parameter  $\alpha$ . When equal to zero there is no hydrogen contribution. When greater than unity the maser will oscillate. Saturation, represented by the factor  $S$ , increases with microwave field amplitude and decreases with absolute signal frequency difference from the hydrogen transition frequency,  $F_0$ .

The complex portion of the hydrogen contribution to  $H_m$  has a ratio of imaginary to real parts which is proportional to the frequency difference  $f - F_0$ . If  $f$  is the frequency of a signal which is intended to equal  $F_0$  then the value of this ratio is proportional to the frequency error. The proportionality factor can be calculated from prior knowledge of the transverse relaxation time,  $T_2$ . For control purposes it need not be known precisely.

FOR THE PASSIVE MASER WITH ATOMIC HYDROGEN:

$$(9) \quad V_1 / V_3 = H_0 H_m \quad \text{WHERE:}$$

$$(10) \quad H_m = 1 + j v_c - (\alpha / (1 + S)) / (1 + j 2\pi T_2 (f - F_0))$$

DERIVED FROM (35), (36) AND (37) OF REFERENCE 1, WHERE

$\alpha$  PARAMETER WHICH CHARACTERIZES OPERATING CONDITIONS  
RELATIVE TO THRESHOLD OF OSCILLATION. (22), REF. 1

$S$  SATURATION FACTOR OF THE ATOMIC TRANSITION (38), REF. 1

$T_2$  TRANSVERSE RELAXATION TIME OF HYDROGEN ATOMS

$F_0$  ATOMIC TRANSITION FREQUENCY

FIGURE 4 MASER INVERSE TRANSFER FUNCTION

Figure 5 introduces the symbol  $H_h$  for the hydrogen contribution to the inverse maser transfer  $H_m$ , and in (12) expresses it in terms of variables  $a_h$  and  $v_h$ . The angle of  $H_h$  is a function only of  $v_h$  (14) which in turn is proportional to  $f - F_0$ .

Let  $f_s$  be the frequency that is to be controlled to equal  $F_0$ , and let  $v_{hs}$  be the value of  $v_h$  at  $f_s$ . Let  $H_{ms}$  be the value of  $H_m$  and  $H_{hs}$  the value of  $H_h$  at frequency  $f_s$ .

Earlier  $H_{ca}$  and  $H_{cb}$  were defined for the cavity without atomic hydrogen. Now we will assume that  $f_a$  and  $f_b$  are sufficiently far from  $F_0$  that the effect of the hydrogen atoms on  $H_m$  at these frequencies is negligible. We showed in Figure 3 that the mean of  $H_{ca}$  and  $H_{cb}$  equals  $H_{cs}$ , which cannot be measured directly in the presence of the atomic hydrogen.

If the computed value of  $H_{cs}$  is subtracted from the maser inverse transfer  $H_{ms}$  the result is  $H_{hs}$  (15). The imaginary part of  $H_{hs}$  divided by its real part gives  $v_h$  (16) which in turn is proportional to the frequency error  $f_s - F_0$ , independent of the value of  $H_{cs}$  which depends on the cavity tuning error  $f_c - f_s$ .

The complex plot of Figure 5 shows three points  $H_{ms}$ ,  $H_{ca}$ , and  $H_{cb}$  representing measured values, and  $H_{cs}$  representing a computed value. The location of  $H_{cs}$  indicates a cavity tuning error  $-0.1$  times the cavity bandwidth ( $2(f_s - f_c)/B_c = 0.2$ ). The line from  $H_{cs}$  to  $H_{ms}$  is horizontal, hence zero imaginary part of  $H_{hs}$ , indicating that  $f_s = F_0$  in this example.

$$(11) \quad H_m = H_c + H_h, \quad \text{WHERE:}$$

$$(12) \quad H_h = -a_h / (1 + j v_h)$$

$$(13) \quad a_h = \alpha / (1 + S)$$

$$(14) \quad v_h = 2\pi T_2 (f - F_0)$$

AT FREQUENCY  $f_s$ ,  $H_m = H_{ms}$  and  $H_h = H_{hs}$  BY DEFINITION

$$(15) \quad H_{hs} = H_{ms} - H_{cs} = H_{ms} - (H_{ca} - H_{cb}) / 2$$

$$(16) \quad -\text{IMAG}(H_{hs}) / \text{REAL}(H_{hs}) = v_h = 2\pi T_2 (f_s - F_0)$$

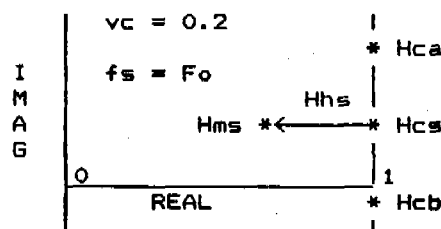


FIGURE 5 HYDROGEN CONTRIBUTION TO MASER INVERSE TRANSFER

## FREQUENCY ERROR COMPUTATION

The frequency error computation would consist of equations (15) and (16) were it not for the angle of  $H_o$ , due to the phase components of the paths which connect the measurement junctions with the maser. For actual measurements the whole plot of Figure 5 would be rotated counterclockwise through the angle of  $H_o$ .

Figure 6 shows relationships which permit computation of the ratio of the real and imaginary components of  $H_{hs}$  from measurements. In the table of Figure 6 we name the measured inverse transfer ratios  $H_s$ ,  $H_a$ , and  $H_b$ , each containing the factor  $H_o$ . The expressions for  $H_{ms}$ ,  $H_{ca}$ , and  $H_{cb}$  are also included.

$H_1$  defined in (17) corresponds to (15), but with the factor  $H_o$  included.  $H_2$  in (18) is the complex value from (5) of Figure 3 with the sign of its imaginary part reversed. Multiplying  $H_1$  and  $H_2$  (19) then produces  $H_{hs}$  multiplied by two real numbers and rotated through  $-\pi/2$ . The real factors are  $4 F_1 / B_c$  and  $(H_o \text{ CONJUGATE } (H_o))$ .

The value of  $v_{hs}$  in terms of the components of  $H_{hs}$  is reproduced in (20), and the equivalent relationship in terms of components of  $H_3$  is given in (21). The frequency error computation in the presence of  $H_o$  consists of (17), (18), (19), and (21).

FREQUENCY	$V_1 / V_3$	H
$f_s$	$H_s = H_o H_{ms}$	$H_{ms} = H_{cs} - a_h / (1 + j v_{hs})$
$f_a = f_s + F_1$	$H_a = H_o H_{ca}$	$H_{ca} = 1 + j 2 (f_s + F_1 - f_c) / B_c$
$f_b = f_s - F_1$	$H_b = H_o H_{cb}$	$H_{cb} = 1 + j 2 (f_s - F_1 - f_c) / B_c$

$$(17) \quad H_1 = H_s - (H_a + H_b)/2 = H_o H_{ms} - H_o H_{cs} = H_o H_{hs}$$

$$(18) \quad H_2 = \text{CONJUGATE } (H_a - H_b) = \text{CONJUGATE } (j (4 F_1 / B_c) H_o)$$

$$(19) \quad H_3 = H_1 H_2 = -j H_{hs} (4 F_1 / B_c) (H_o \text{ CONJUGATE } (H_o))$$

$$(20) \quad v_{hs} = -\text{IMAG } (H_{hs}) / \text{REAL } (H_{hs})$$

$$(21) \quad 2\pi T_2 (f_s - F_o) = \text{REAL } (H_3) / \text{IMAG } (H_3)$$

FIGURE 6 SOLUTION FOR SIGNAL FREQUENCY ERROR

## DISCUSSION

The signal processing scheme described here for reducing the cavity pulling factor of a passive maser appears to offer the following:

1. Reduction of errors due to uncompensated cavity resonance variations (temperature, etc.)
2. Reduction of errors due to receiver noise and electronic system imperfections in the cavity servo
3. The possibility of operating with a temperature-stable cavity without autotuning.

The method may also be of use in monitoring the cavity drift in large active masers without autotuning.

The method has apparent limitations. The frequency-error computation is based on six measured real values (three complex values) compared to three in the case of criterion (c), and two in criterion (a). Each of these values includes a contribution due to receiver noise. Noise analysis for an oscillator control servo using this method has not yet been accomplished. Some rough reasoning indicates that the frequency-error noise density will be greater than for criterion (c).

The model assumes a single-mode cavity resonator with ideal transfer function symmetry. Sensitivity to unwanted cavity modes, non-ideal microwave circuits, and filters in the common signal path is not known. Additional circuit transfer function elements may be accommodated by taking measurements at additional frequencies, but with the penalty of additional noise.

The hydrogen influence at the side frequencies  $f_a$  and  $f_b$  was neglected in the derivations. The real part of this influence is less than the square of  $1/v_h$  evaluated at the side frequency. The absolute value of the imaginary part is less than  $1/v_h$ , closely equal and opposite at the two frequencies. In the absence of saturation these would cause no errors. At actual operating levels they will cause higher order pulling, showing up for sufficiently large cavity tuning error,  $v_{cs}$ . The error  $v_{cs} = 0.2$  in Figure 5 is for illustration but is undoubtedly far too large for satisfactory accuracy of the assumption.

While it is not clear whether this error-detection scheme is advantageous, the derivations imply that cavity pulling is not an unavoidable perturbation of the atomic hydrogen emission but is a result of the scheme used to approximate its frequency.

#### ACKNOWLEDGEMENTS

The author's familiarity with the properties of maser transfer functions is a direct result of his studies in maser electronics for the Naval Research Laboratory, particularly those concerned with the phase-sensing servos in their passive masers. These studies were carried out under the direction of Joe White, with much measurement and computer support from Alick Frank and Al Gifford. The above scheme was developed independently.

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